Author's Reply to Comment by Kenneth Wang

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THE solution by Wang and Ting[1] was obtained by assuming V, contained in the term $[(gR_0/V^2) - 1]$, equal to the initial entry speed V_f, which is a constant. After initial portion of entry [say, $V/(gR_0)^{1/2}$ less than 1], V is quite different from the initial entry speed V_f; consequently, the solution no longer applies. The numerical values (0.986 and 10°32′) calculated by Wang[2] agree well with both the second-order solution[3] and the exact numerical solution, Figs. 8c and 8d, but they are for "from entry to skip," which is the initial portion of the overall re-entry trajectory dealt as a whole by the second-order theory.[3] When the second-order solution applies only to the initial portion of the overall re-entry trajectory [say, for $V/(gR_0)^{1/2} = 2^{1/2}$ to 1 or approximately 1], the second-order solution reduces to Wang's solution.[4]

References

[1] Wang, K. and Ting, L., "An approximate analytical solution of re-entry trajectory with aerodynamic forces," ARS J. **30**, 565–566 (1960).

[2] Wang, K., "Comment on 'A second-order theory of entry mechanics into a planetary atmosphere'," AIAA J. **1**, 977 (1963).

[3] Loh, W. H. T., "A second-order theory of entry mechanics into a planetary atmosphere," J. Aerospace Sci. **29**, 1210–1221 (1962).

[4] Loh, W. H. T., "Supercircular gliding entry," ARS J. **32**, 1398 (1962).

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Comment on "Stability of Pressure Waves in a Combustion Field"

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IN his recent technical note, Rosen[1] made an illuminating and welcome contribution to the field of combustion instability. However, several points are worthy of clarification. It is assumed in this note that the physical and chemical combustion processes are represented by a one-step rate controlling reaction that is a function of chamber conditions at the instant of reaction. It should be noted, however, that finite time delays occur and are important in actual combustion chambers. Although it is true that simple chemical reaction rates may follow such a one-step, instantaneous law, other processes such as mixing, vaporization, or a complex chemical reaction do not. When the characteristic times of these more complex processes are of the same order as the wave propagation time, important interactions occur.

After a transformation to a Lagrangian coordinate system, the author states that the linearity of the governing differential equation for the pressure outside of the combustion zone indicates that the pressure waves "propagate in a trivial way without growth, decay, or dispersion." This usually is not the case for ordinary gases. The linearity of the differential equations demands that the acoustic impedance (which equals the square root of minus the partial derivative of pressure with respect to specific volume at constant entropy) be constant with pressure or volume. This demand has been satisfied by the choice of the equation of state. This equation of state never is true for polytropic gases and is true only for such materials as solids which obey Hooke's Law. A realistic choice of a state equation would have shown a distortion of the waveform due to nonlinear effects and thus a possible shock formation. Once the shock has formed, one cannot say there is "no growth, decay, or dispersion." For example, the asymptotic behavior of the N wave is well known to be such that the shock strength decreases in inverse proportion to the square root of time, and the width of the wave increases as the square root of time.[2] Also, note that care must be taken in applying the transformation when a shock or detonation discontinuity is present, since the Jacobian of the transformation becomes discontinuous at such points. The transformation must be applied separately on each side of the discontinuity, and then the coordinates should be matched at the point of discontinuity.

Finally, and most important, the author has implied that, although boundary conditions may affect the stability criteria for certain practical configurations, the local interaction between a pressure wave and the combustion process is most important in determining the stability criteria. This generally is not the case, since dissipation phenomena introduced through the boundary conditions usually are as important as the forcing function introduced through the combustion process. For instance, in liquid rocket engine instability, there is a loss of oscillation energy by convection of the mean flow out the nozzle. Furthermore, oscillation impedance of the nozzle usually causes a reflected wave to have a lower energy than the incoming wave. Both of these effects are of the same importance as the forcing function, which is of the order of the mean flow. Therefore, in practical problems, one cannot look at local stability criteria but must concern himself with stability in the large.

References

[1] Rosen, G., "Stability of pressure waves in a combustion field," ARS J. **32**, 1605–1607 (1962).

[2] Courant, R. and Friedrichs, K. O., *Supersonic Flow and Shock Waves* (Interscience Publishers Inc., New York, 1948), pp. 164–168.

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Reply by Author to W. C. Strahle and W. A. Sirignano

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THE preceding comment by Strahle and Sirignano discusses two aspects of Ref. 1. Some practical limitations of the idealized mathematical model are mentioned. It then is asserted that local stability criteria should not be studied and that the stability problem is always essentially global in character, like a boundary-value or eigenvalue problem.

With regard to the practical limitations of the model, it should be pointed out that a more general form for the

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process relation [Eq. (4)]† in place of the tangent-gas law [Eq. (9)] brings in additional nonlinearity that has trivial consequences, if compared to the nonlinear interaction between pressure waves and the combustion processes. Although the equations become less transparent, conclusions derived from the mathematical model are not modified essentially if Eq. (9) is replaced by a polytropic process relation.‡ Therefore, in order to make new and interesting phenomena clearly evident, convective nonlinearity is suppressed in the theoretical treatment[1] by evoking the tangent-gas law. Emphasizing the shortcomings of a burning rate function like Eq. (8), a representation of the combustion mechanism which takes account of vaporization and chemical reaction but cannot take account of physical mixing and molecular diffusion (processes that are rate-controlling in many combustion fields) is earnest criticism. By introducing a time delay function in place of Eq. (8), by linearizing the system of equations, and by supplementing the equations that result with boundary conditions, it is possible to obtain a resonance-type mechanism for instability, a mechanism that does not appear in the idealized model of Ref. 1. Yet the issue is not whether a representation of the combustion mechanism like Eq. (8) is of completely general practical interest, but rather whether it is generally less appropriate than the ad hoc introduction of a time delay function. It certainly is interesting that a mechanism for instability exists even if processes associated with a time delay (such as physical mixing and molecular diffusion) are not rate-controlling. In fact, this "friction term" interaction between pressure waves and the combustion processes must appear in an all-embracing mathematical theory, one with additional equations for mixing and diffusion, because the idealized mathematical model in Ref. 1 is the limit of such a theory with "time delays" permitted to vanish. Furthermore, it is questionable whether a resonance-type mechanism for instability can be derived in rigorous fashion from the complete set of governing equations in an all-embracing mathematical theory.§ No one can argue that the complicated and variegated combustion fields of practical interest are not worthy of analysis from complementary points of view.

For practical situations that are described approximately by the idealized model, consider whether the mechanism for instability is essentially local in character, as asserted in Ref. 1. Provided that it is appropriate to relate the idealized mathematical model to an actual combustion field, Eq. (19)

$$\frac{1}{2}(1+\omega_0)a\frac{\partial^2 P}{\partial t^2} + \bar{\phi}[a(m+1)P^{m-1} - b(m-1)P^{m-2}] \times \frac{\partial P}{\partial t} - \frac{\partial^2 P}{\partial \psi^2} = 0$$

governs the dynamics of pressure waves in the region of active combustion. Observe that the equation has the deflagration solution $P \equiv P_c = \text{const}$, with P_c interpreted physically as the average "chamber pressure" for steady, normal combustion. In the neighborhood of the $P \equiv P_c$ solution, the equation has the general acoustical perturbative solution

$$\frac{P}{P_c} = 1 + \sum_n \epsilon_n e^{-\mu_n t}\sin(\nu_n t + \delta_n) \times \sin\left(k_n\left[\frac{(1+\omega_0)a}{2}\right]^{1/2}\psi + \theta_n\right)$$

where the ϵ's are arbitrary small constants, $\epsilon_n^2 \ll 1$, the k's denote wave-numbers in ψ space for acoustical disturbances that are consistent with conditions at a remote boundary (well outside the region of active combustion), the δ's and θ's denote phase constants that are prescribed by the same boundary conditions, and the μ's and ν's are given by

$$\left.\begin{aligned}\mu_n &= \chi \pm (\chi^2 - k_n^2)^{1/2}\\ \nu_n &= 0\end{aligned}\right\} \qquad [k_n^2 \leqslant \chi^2]$$

$$\left.\begin{aligned}\mu_n &= \chi\\ \nu_n &= \pm (k_n^2 - \chi^2)^{1/2}\end{aligned}\right\} \qquad [k_n^2 \geqslant \chi^2]$$

with the abbreviation

$$\chi \equiv \frac{\bar{\phi}[a(m+1)P_c^{m-1} - b(m-1)P_c^{m-2}]}{(1+\omega_0)a}$$

Clearly, the $P \equiv P_c$ solution is stable if all of the μ's are positive and unstable if any μ_n is negative. But, independent of the magnitude of each admissible wave number k_n (or phase constants δ_n and θ_n), each μ_n is positive (negative) if the parameter χ is positive (negative). Thus the stability depends entirely on the sign of χ and does not depend on boundary conditions that fix the k's, the δ's, and the θ's. Hence the mechanism for instability is essentially local in character. The condition for stability, namely, $\chi > 0$, is recast in a neat form by introducing the effective polytropic index, as by Eq. (27):

$$\kappa = (b/aP_c) - 1$$

Then the necessary and sufficient condition for pressure wave stability is obtained as‖

$$m < [1 + (2/\kappa)]$$

a result derived by alternative considerations and reported in Ref. 1.

† Equations quoted are in Ref. 1.

‡ For example, the necessary and sufficient condition for stable pressure waves $\{m < [1 + (2/\kappa)]\}$ still is obtained if one replaces Eq. (9) with a polytropic process relation, as one may deduce easily from the results in Ref. 2.

§ Attempts by the author to obtain theoretical results of this nature were not fruitful.

References

[1] Rosen, G., "Stability of pressure waves in a combustion field," ARS J. **32**, 1605–1607 (1962).

[2] Rosen, G., "Nonlinear pressure oscillations in a combustion field," ARS J. **30**, 422–423 (1960).

‖ Note that, if this condition for pressure wave stability is satisfied, it is satisfied by only a slim margin, since m ranges between 1 and 3 in practical cases, while $[1 + (2/\kappa)]$ ranges between 2 and 3 for real gases. For larger values of the "chamber pressure" P_c, pressure wave stability is less likely; the rate-controlling physical and/or chemical processes usually are associated with a larger value for m at higher pressures, binary and ternary molecular processes playing a more significant role.

Comments on "Free Vibration of a Damped Elliptical Plate"

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In Ref. 1, results are presented for the two lowest eigenfrequencies of a clamped-edge elliptical plate. It should be pointed out, however, that only one of the eigenfrequencies determined has any physical significance, viz., the solution corresponding to λ_1^2, since the assumed modal form ϕ can-

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